# Structure-Function Relationships in Connectome Echo State Networks

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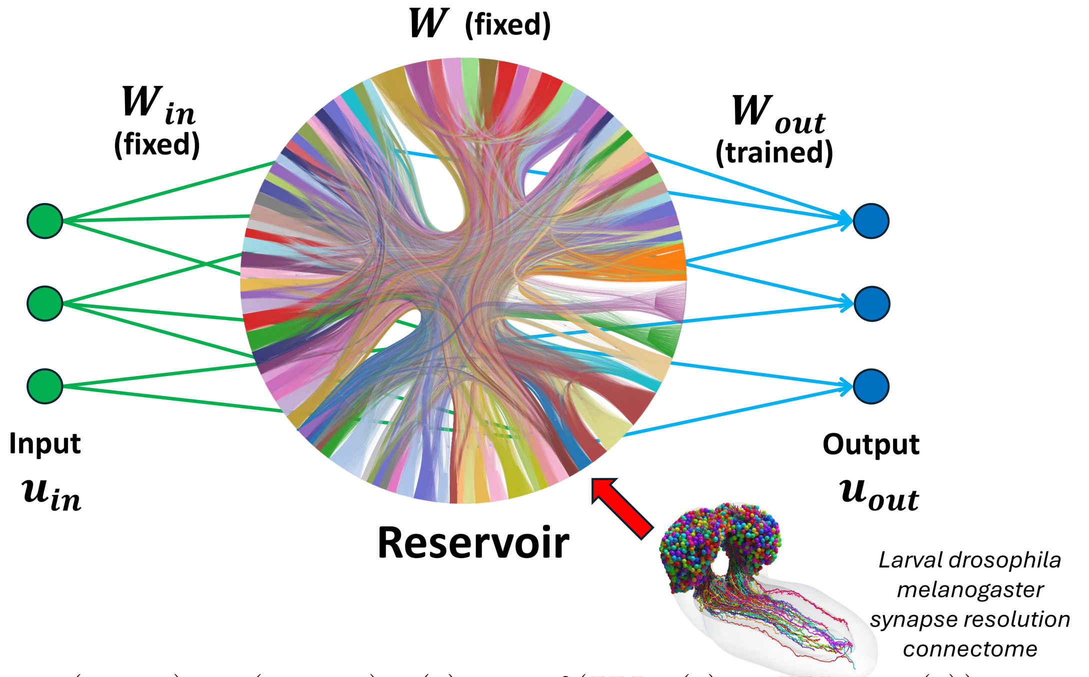
## Introduction

The Echo State Network (ESN) framework [3] is an efficient computational paradigm & has been suggested as a model of brain function [2]. It is unknown how structure influences function & robustness of ESNs. We used biological networks [5] to study this, compared with randomly initialised ESNs.

### Echo State Networks & *Drosophila*

The setup involves an input & recurrent layer, which remain fixed, & an output layer which is trained by linear regression.

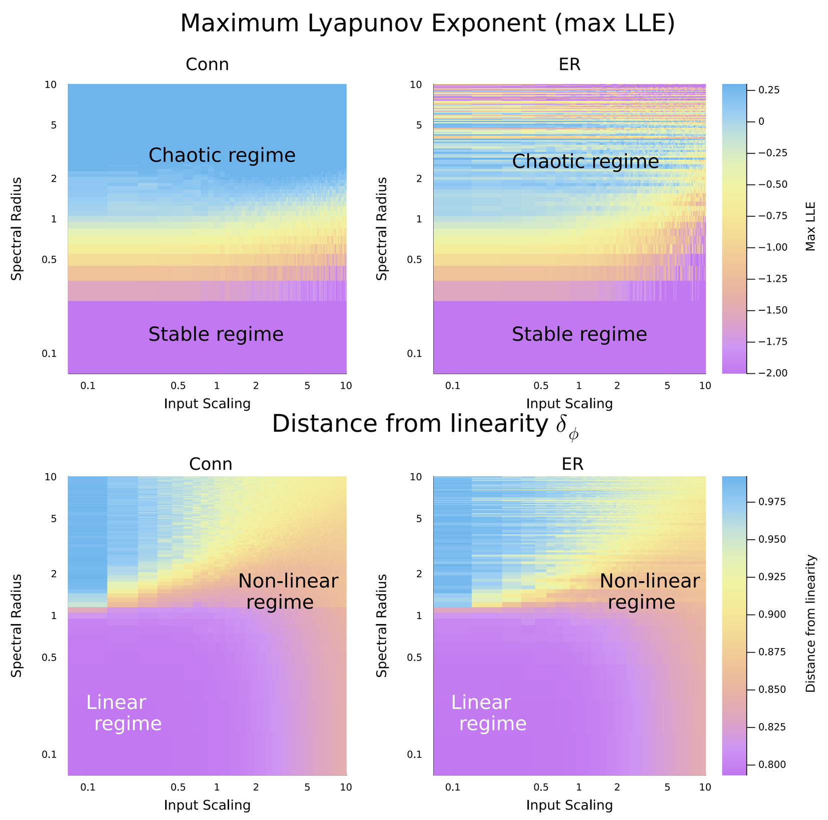



$$\boldsymbol{x}(t+1) = (1-\alpha)\boldsymbol{x}(t) + \alpha f(\boldsymbol{W}\boldsymbol{x}(t) + \boldsymbol{W}_{\text{in}}\boldsymbol{u}_{\text{in}}(t))$$

We used a hierarchical stochastic block model [4, 1] to infer communities in the larval *Drosophila melanogaster* Connectome (Conn). These subnetworks were used as bases for ESNs. For comparison we generated equivalent random Erdős-Rényi (ER) & Configuration model (CFG) ESNs.

## Dynamical Regimes

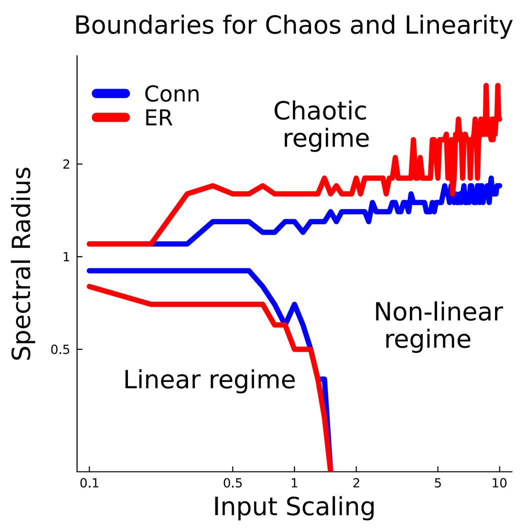
**Max Lyapunov Exponent** measures chaotic behaviour. Operating near the edge of chaos allows balancing memory capacity & non-linear computation. **Distance from linearity** quantifies the degree of the non-linear transformation in the ESN.



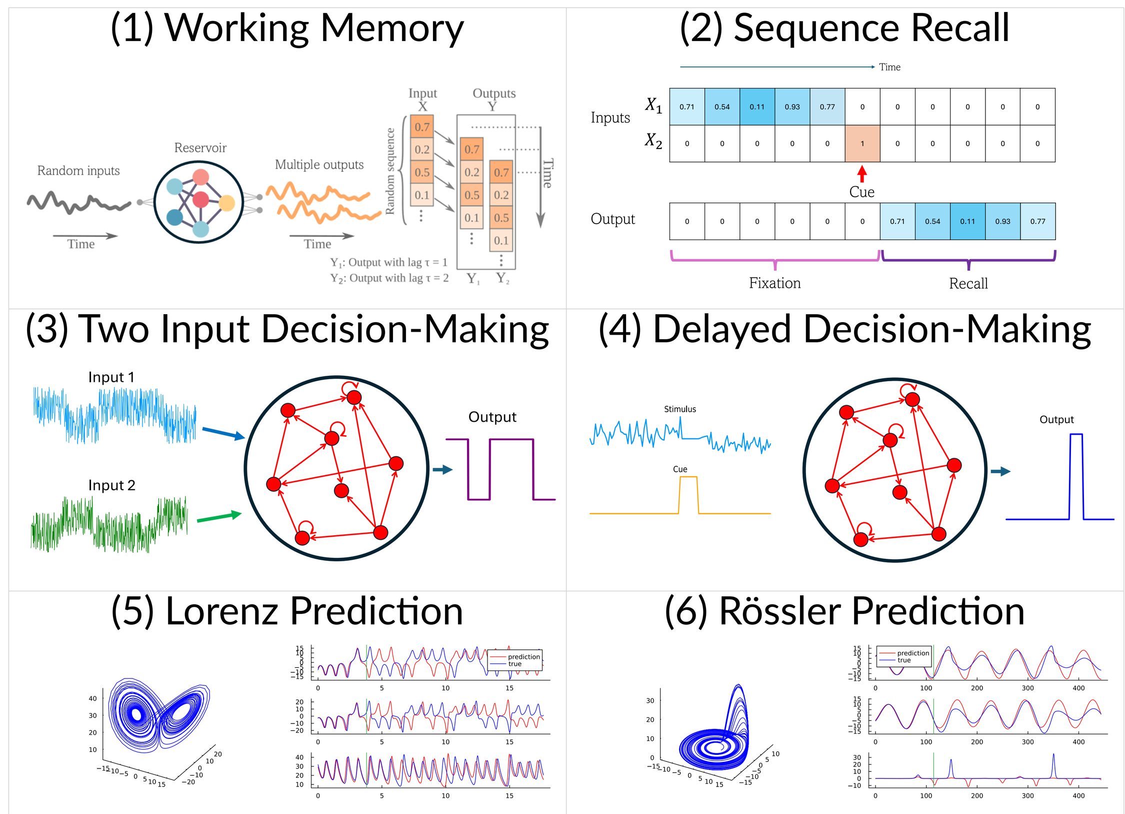

$$LLE_i = \log\left(\prod_t^T |\lambda_i[k]|^{1/T}\right)$$

where $\lambda_i[k]$ is the $i$th eigenvalue of the Jacobian of the ESN at time $k$.

$\delta\phi = 1 - \frac{E_c}{E_{\text{tot}}}$ where $E_c, E_{\text{tot}}$ denote the energies of the Fast Fourier Transform of the input signal & the ESN.




## Computational Tasks




### Task Performances

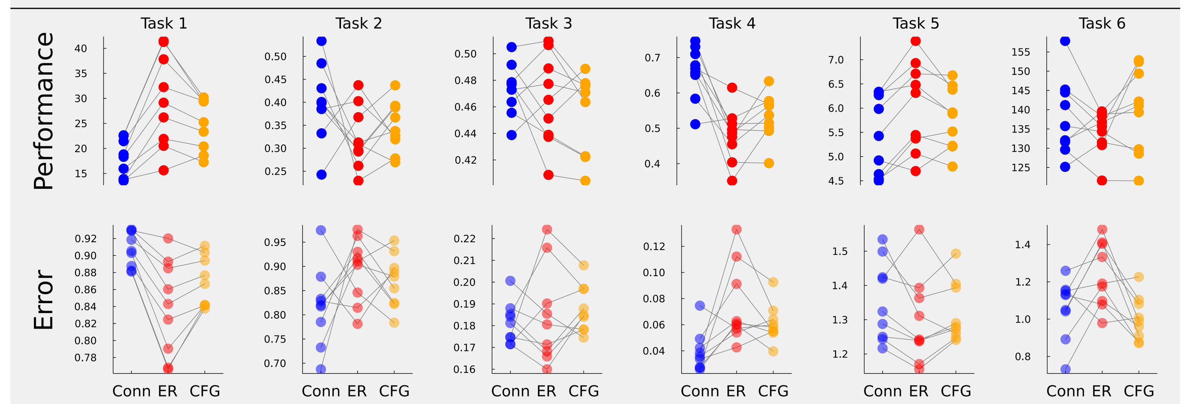


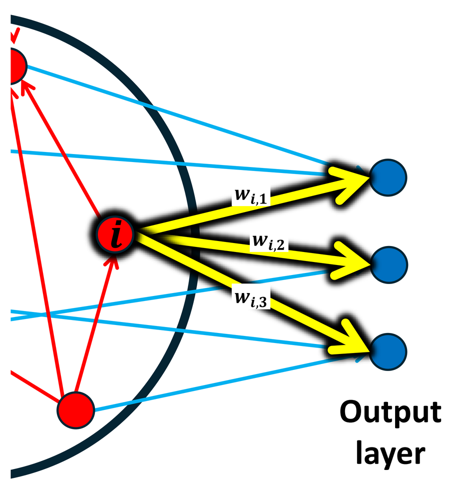

Figure 1. Performances & errors for the 6 tasks across the 9 subnetworks.

## Measuring the Sparsity of Neural Engagement

$$WTV_i(A) := \left(\sum_{w_{i,z}\in W_{out}} |w_{i,z}|\right)_j \langle [r_i(j,t) - \langle r_i(j',t)\rangle_{j'}]^2\rangle_{j,t}$$

Weighted Task Variance (WTV) of neuron $i$ on Task $A$ measures the weighted contribution & engagement of a neuron during an activity. Participation ratio (PR) measures how evenly distributed a quantity is:

$$PR = \frac{(\sum_i x_i)^2}{\sum_i x_i^2}$$




We calculated mean WTV participation across the subnetworks & tasks:

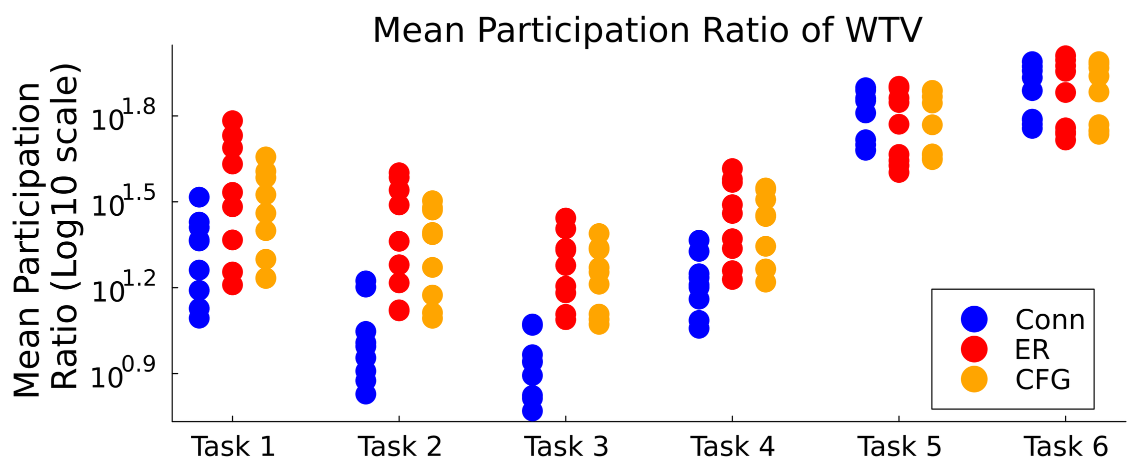
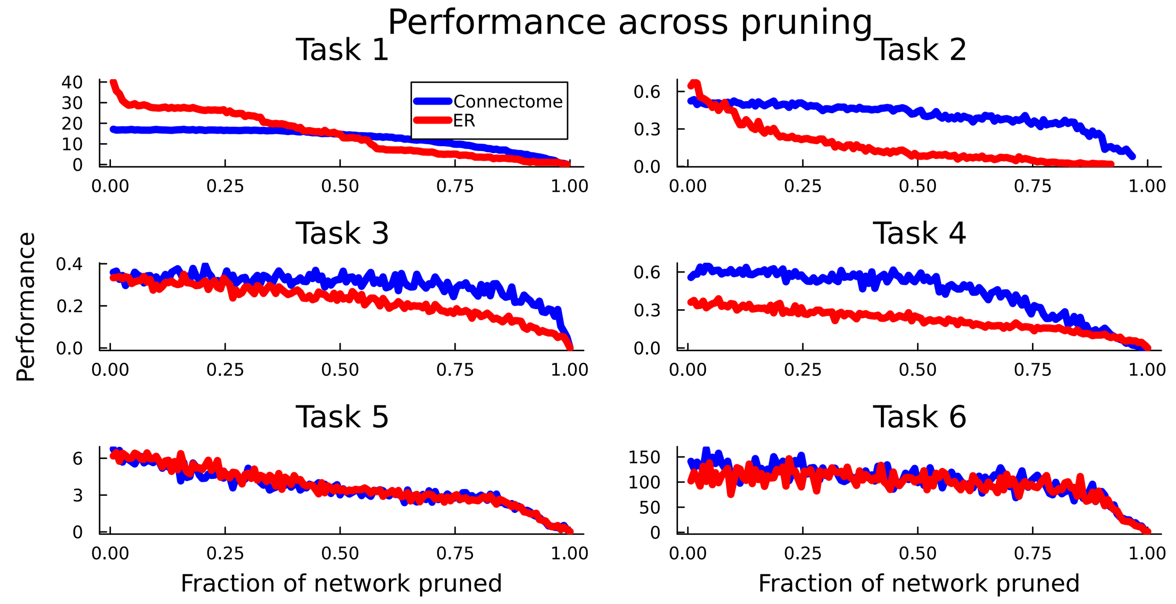



Figure 2. Mean Participation Ratio of WTV across 9 subnetworks & 6 tasks.

## Pruning Nodes from Networks

- The Conn ESNs have lower participation ratio of WTV in Tasks 1–4, suggesting more **sparse** neural engagement, while that of ER & CFG ESNs is more distributed.
- We checked this by **pruning** nodes from the networks in order of increasing WTV.



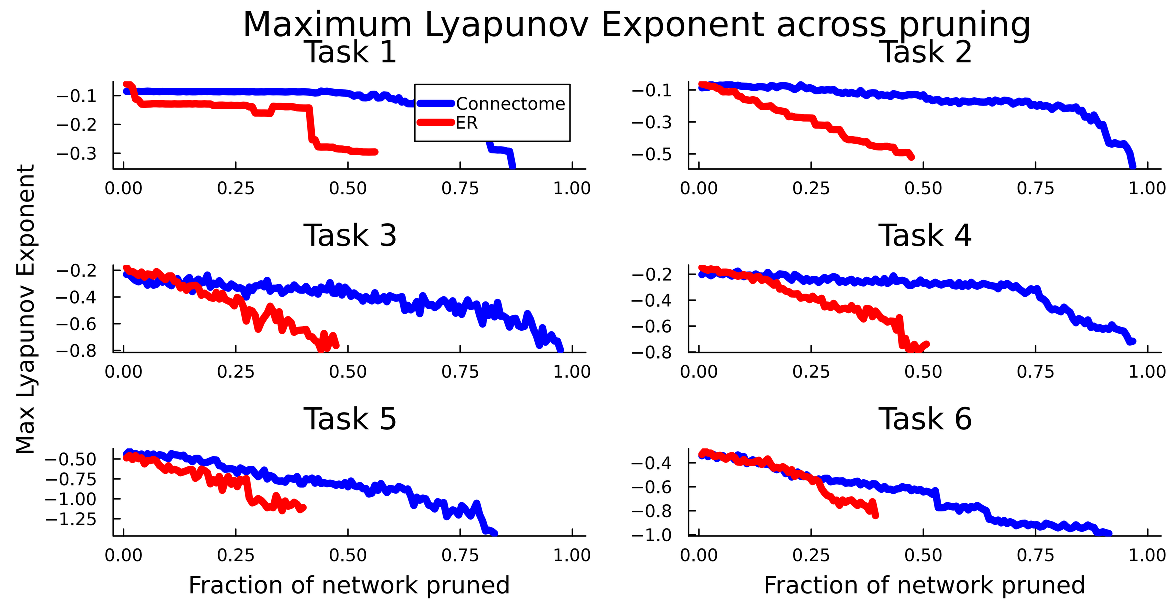




Figure 3. Performance & criticality for example subnetwork across pruning.

## Network Structure

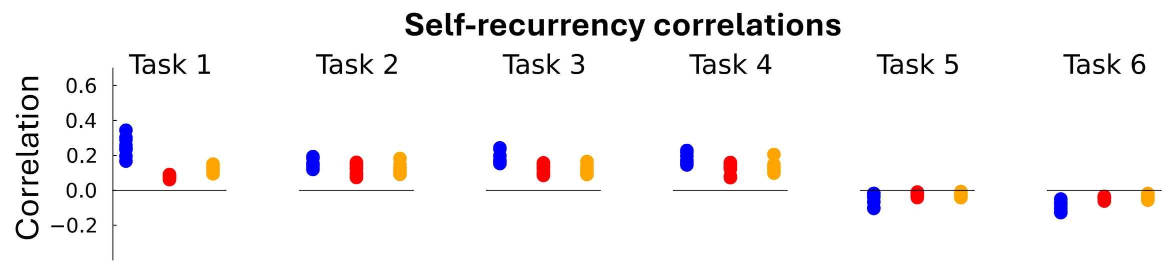



Figure 4. Correlations between self-recurrency & WTV across all 9 subnetworks.

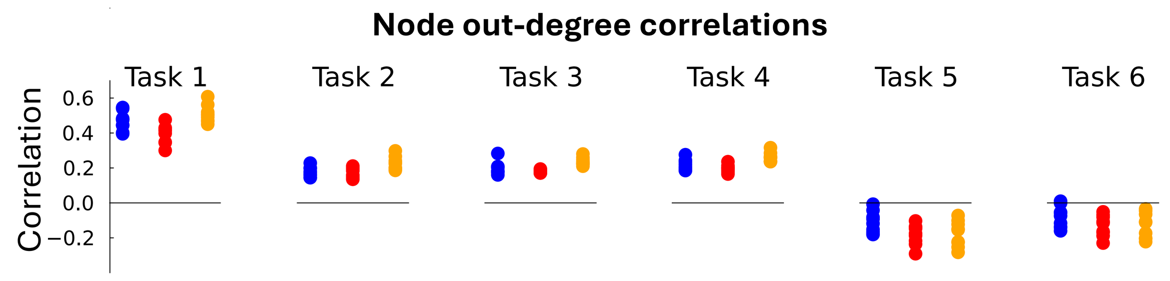



Figure 5. Correlations between node out-degree & WTV across 9 subnetworks.

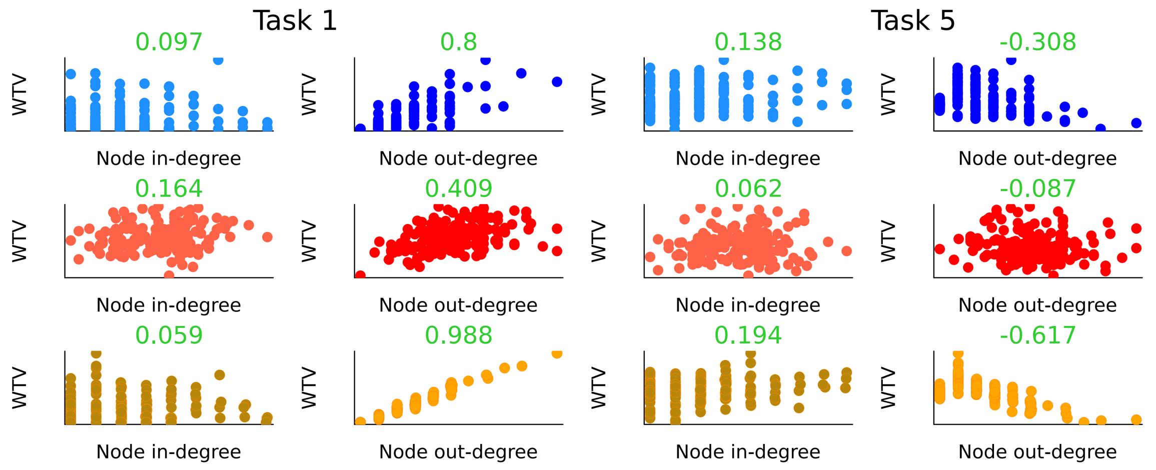



Figure 6. Node degree vs WTV (from one example subnetwork and Tasks 1 & 5).

## Connectome Neuron Annotations

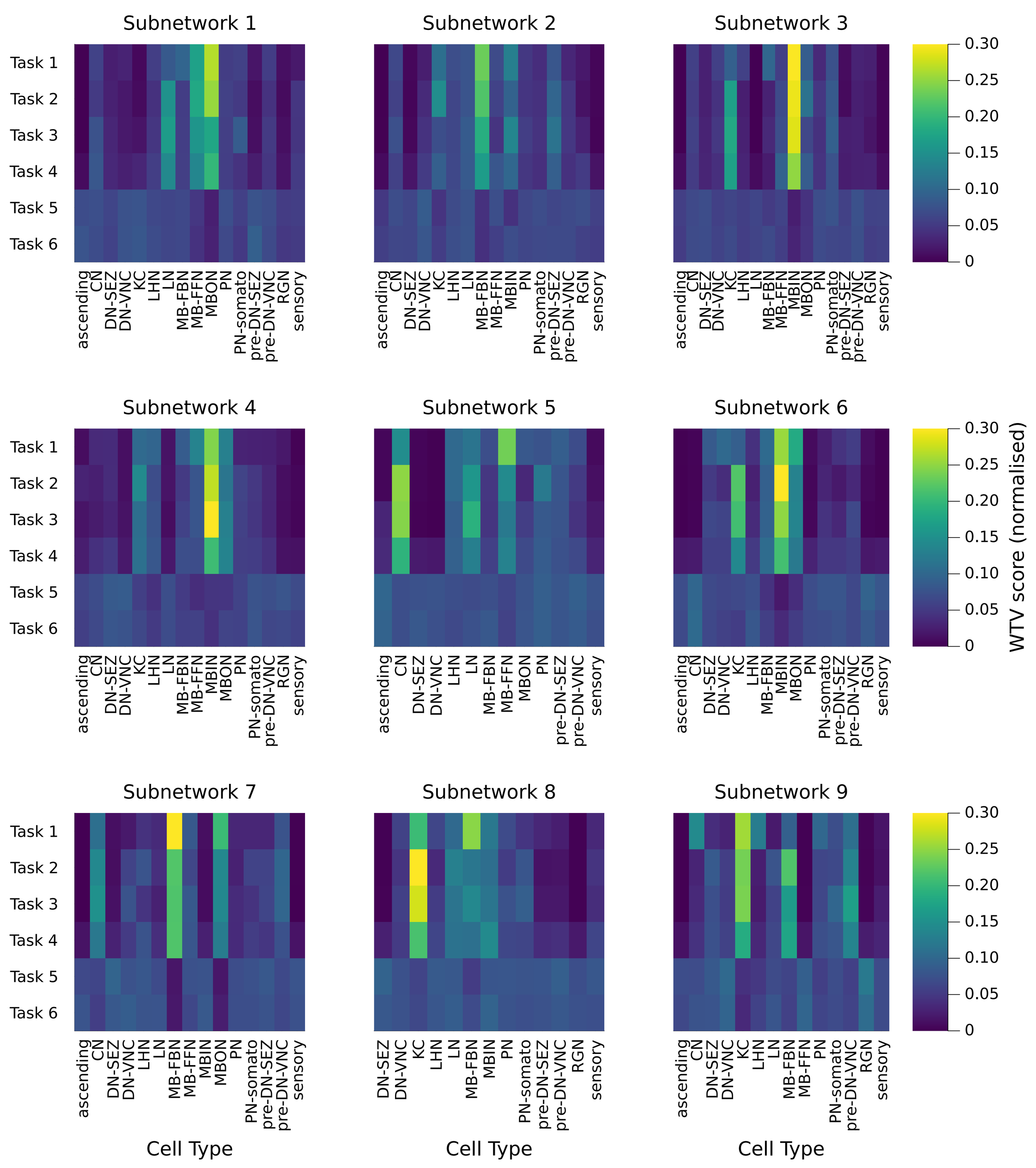



Figure 7. Relative importance score of different cell types from the connectome.

## Conclusions

- Conn topologies yield ESNs with dynamical regimes that vary from conventional ESNs, with differing boundaries between chaos, linearity, & non-linearity.
- The task performances of Conn ESNs are comparable (other than memory) to conventional networks.
- Conn ESNs exhibit a more **sparse neural engagement**.
- We checked if this suggests efficiency & robustness by pruning nodes.
- Identifying **structural features** (such as reciprocity, node degree, & biological annotation) linked to neural contribution points out a potential way of generating more efficient, robust, & task-specific networks.

## Future Work

We aim to use the structural & biological characteristics we have linked with WTV, sparseness, and task specificity to generate networks with these properties enhanced. We want to see if we can use the insights here to initialise better performing, more efficient and robust ESNs.

## References

[1] R. Betzel, M. G. Puxeddu, and C. Seguin. "Hierarchical communities in the larval Drosophila connectome: Links to cellular annotations and network topology". In: *Proceedings of the National Academy of Sciences* 121.38 (2024). [2] P. F. Dominey. "Cortico-striatal origins of reservoir computing, mixed selectivity, and higher cognitive function". In: *Reservoir Computing: Theory, Physical Implementations, and Applications* (2021). [3] H. Jaeger. "Adaptive nonlinear system identification with Echo state networks". In: *Proceedings of the 16th International Conference on Neural Information Processing Systems*. NIPS'02. Cambridge, MA, USA: MIT Press, 2002. [4] T. P. Peixoto. "The graph-tool python library". In: *figshare* (2014). [5] M. Winding et al. "The connectome of an insect brain". In: *Science* 379.6636 (2023).